

Exercise 35

Use the Squeeze Theorem to show that $\lim_{x \rightarrow 0}(x^2 \cos 20\pi x) = 0$. Illustrate by graphing the functions $f(x) = -x^2$, $g(x) = x^2 \cos 20\pi x$, and $h(x) = x^2$ on the same screen.

Solution

Since

$$-x^2 \leq x^2 \cos 20\pi x \leq x^2$$

and

$$\lim_{x \rightarrow 0}(-x^2) = \lim_{x \rightarrow 0}(x^2) = 0,$$

then

$$\lim_{x \rightarrow 0} x^2 \cos 20\pi x = 0$$

by the squeeze theorem. The graph below of each of the functions versus x confirms this result.

